2/11/23

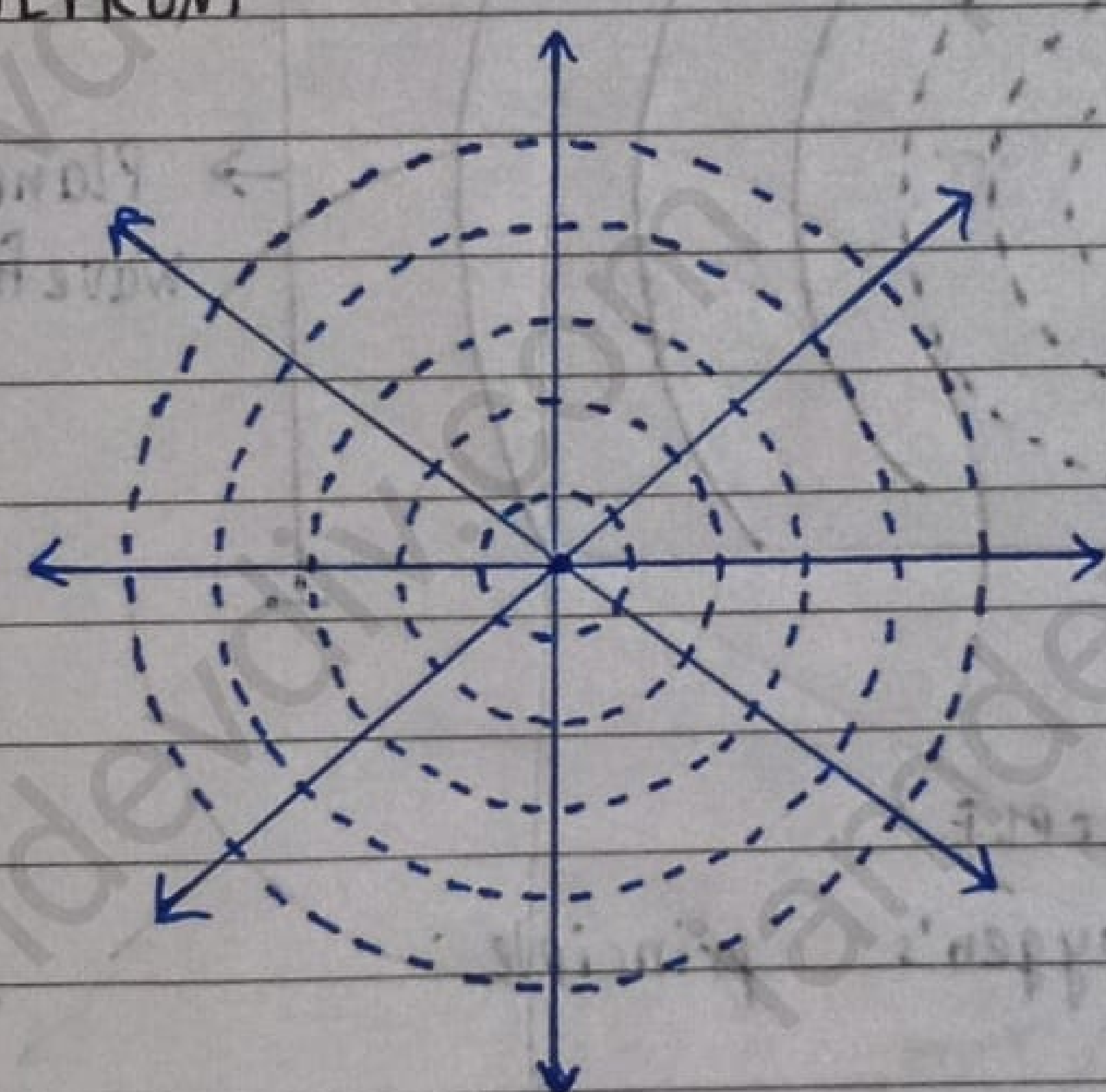
CHAPTER-10
WAVE OPTICS

☆ WAVE FRONT

Wave front is group of all the points of ~~wave all the poin~~ wave having same phase.

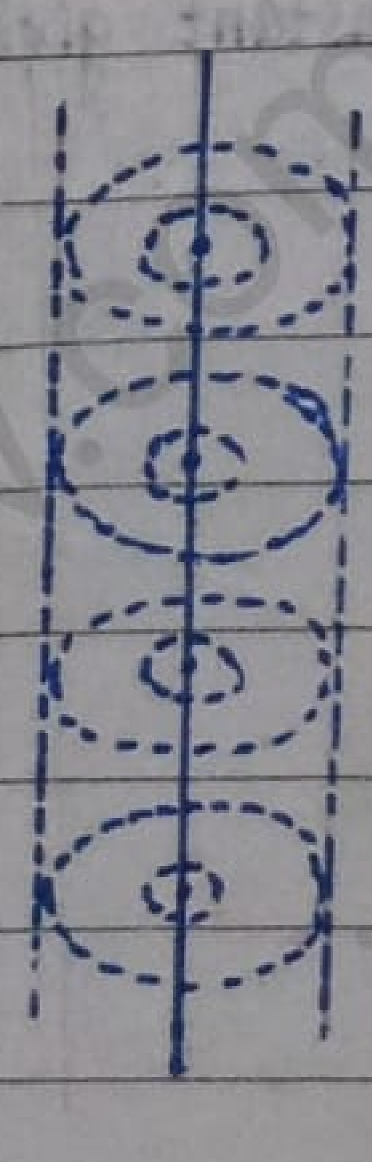
• TYPES OF WAVEFRONT

1. SPHERICAL WAVEFRONT



When a source of Light is a point source then the wave front is called spherical wave front.

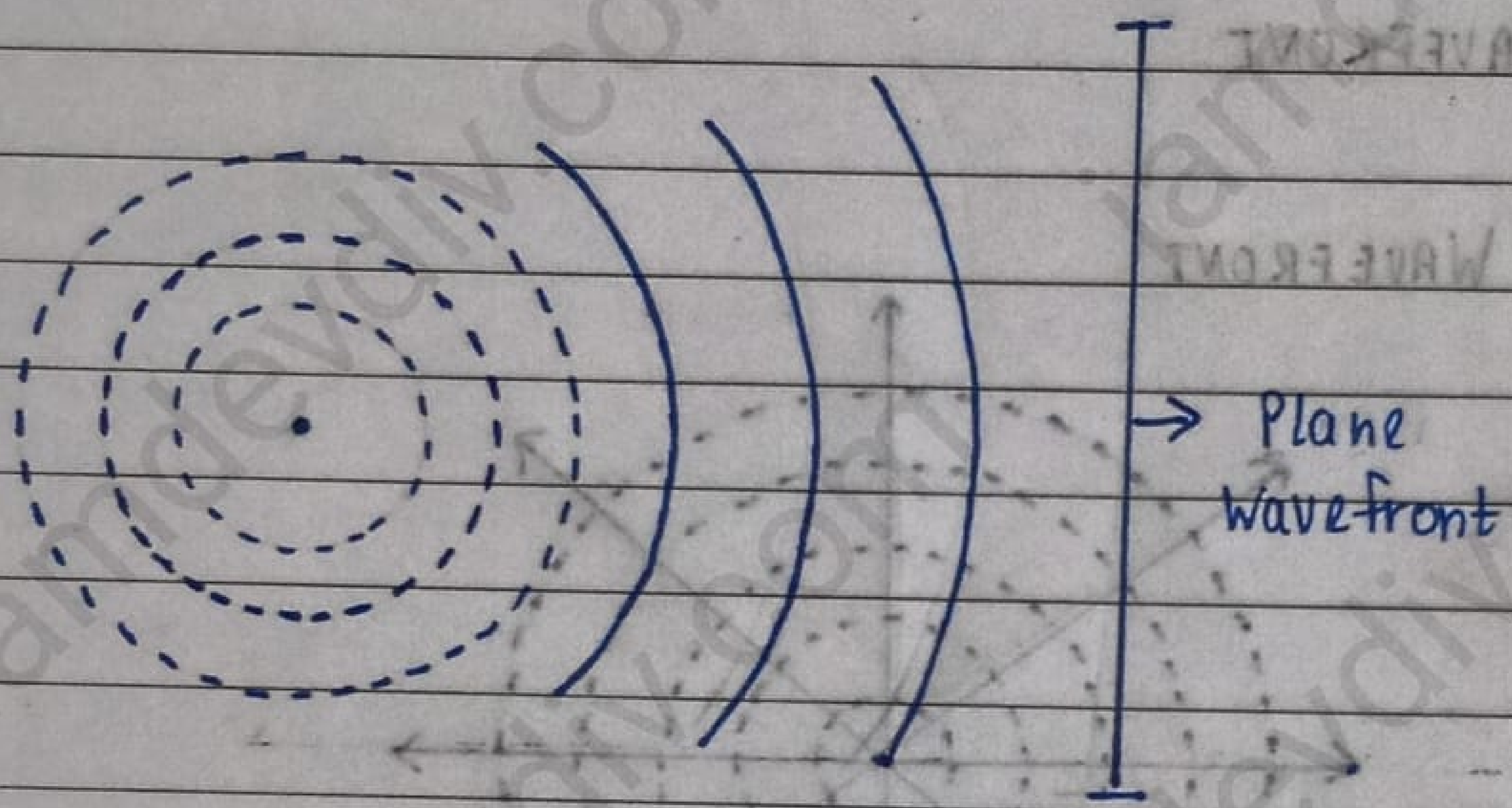
2. CYLINDERICAL WAVEFRONT



When the source of light is linear, example: a slit; then cylinderical wavefront is formed.

3. PLANE WAVEFRONT

When the source of light is at very Large distance then such a wave front is called plane wave front.

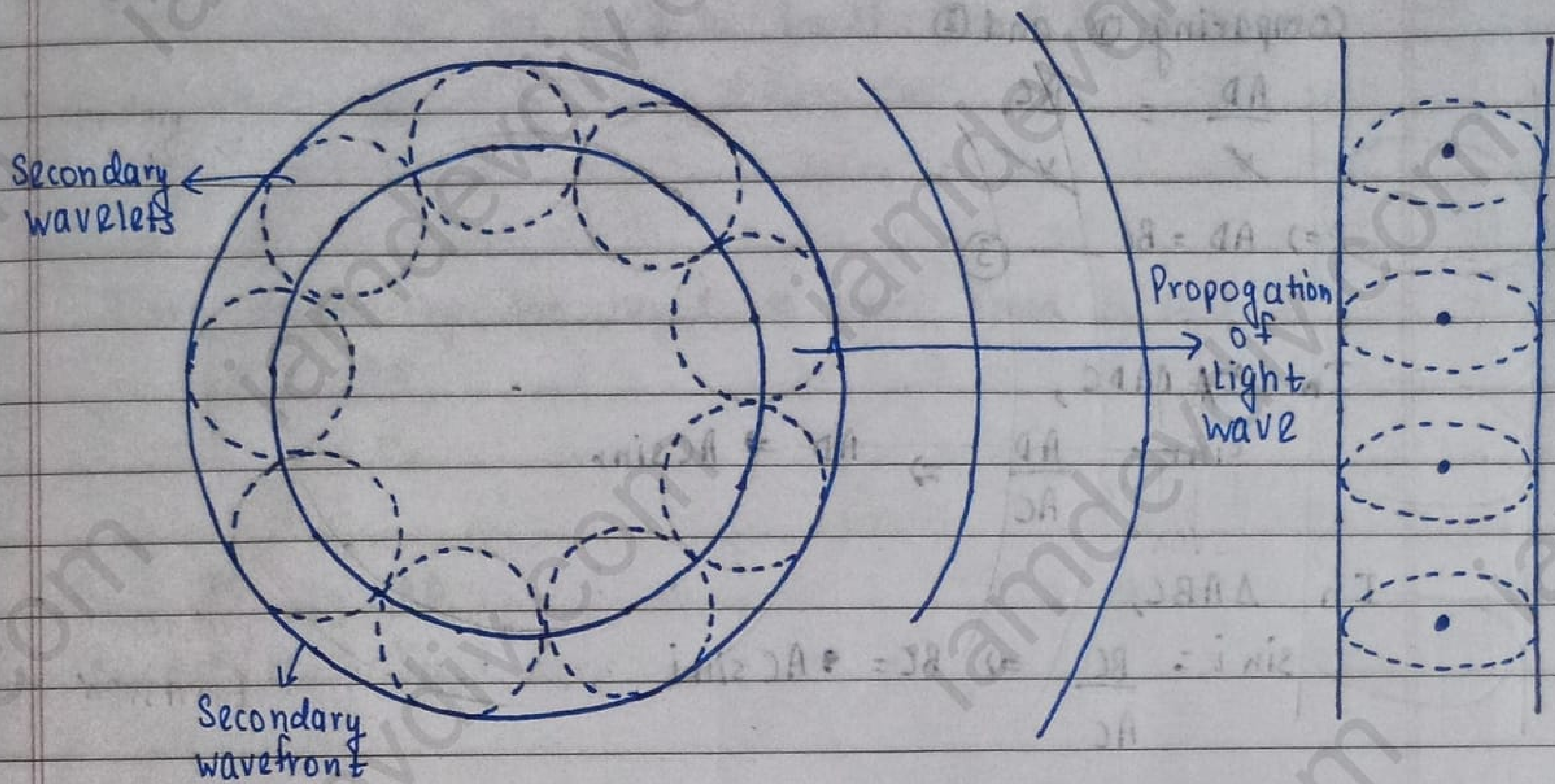


★ HUYGEN'S PRINCIPLE

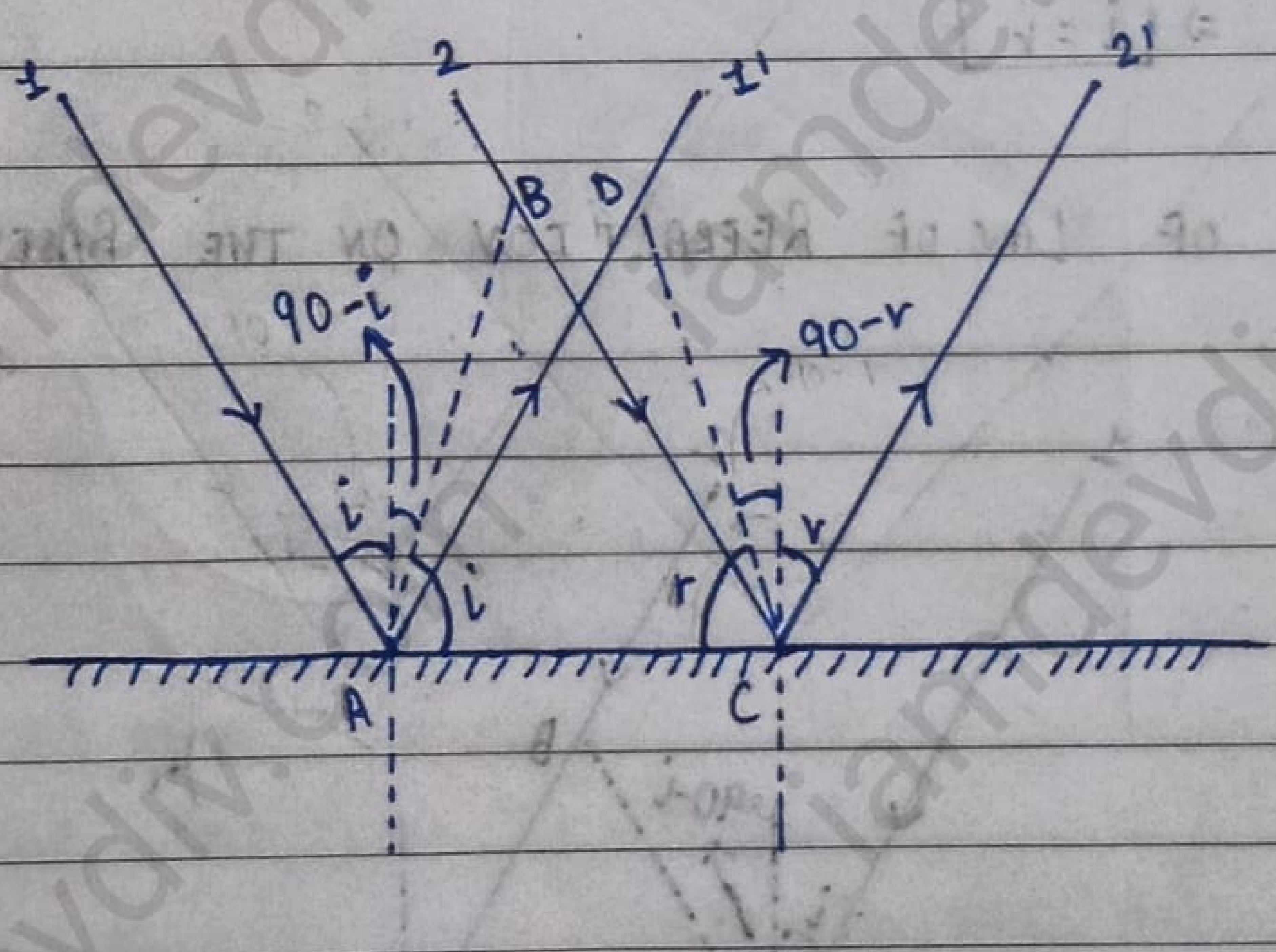
According to Huygen's principle:

1. Every point on the given wave front acts as fresh source of new disturbance called secondary wavelets, which are travelling in all the directions with the velocity of light in the medium.

2. The surface touching these secondary wavelets tangentially in the forward direction at any instant gives new wave front which are called secondary wavefront.



★ REFLECTION ON THE BASIS OF WAVE THEORY



Let 1 and 2 are incident ray and 1' and 2' are refractive rays. If v is the velocity of light, t is the time taken by light to go from B to C or A to D:

$$t = \frac{AD}{v} \quad \text{—— ①}$$

$$t = \frac{BC}{v} \quad \text{—— ②}$$

Comparing ① and ②

$$\frac{AD}{\cancel{v}} = \frac{BC}{\cancel{v}}$$

$\Rightarrow AD = BC$ —— ③

In ΔADC,

$$\sin r = \frac{AD}{AC} \Rightarrow AD = AC \sin r$$

In ΔABC,

$$\sin i = \frac{BC}{AC} \Rightarrow BC = AC \sin i$$

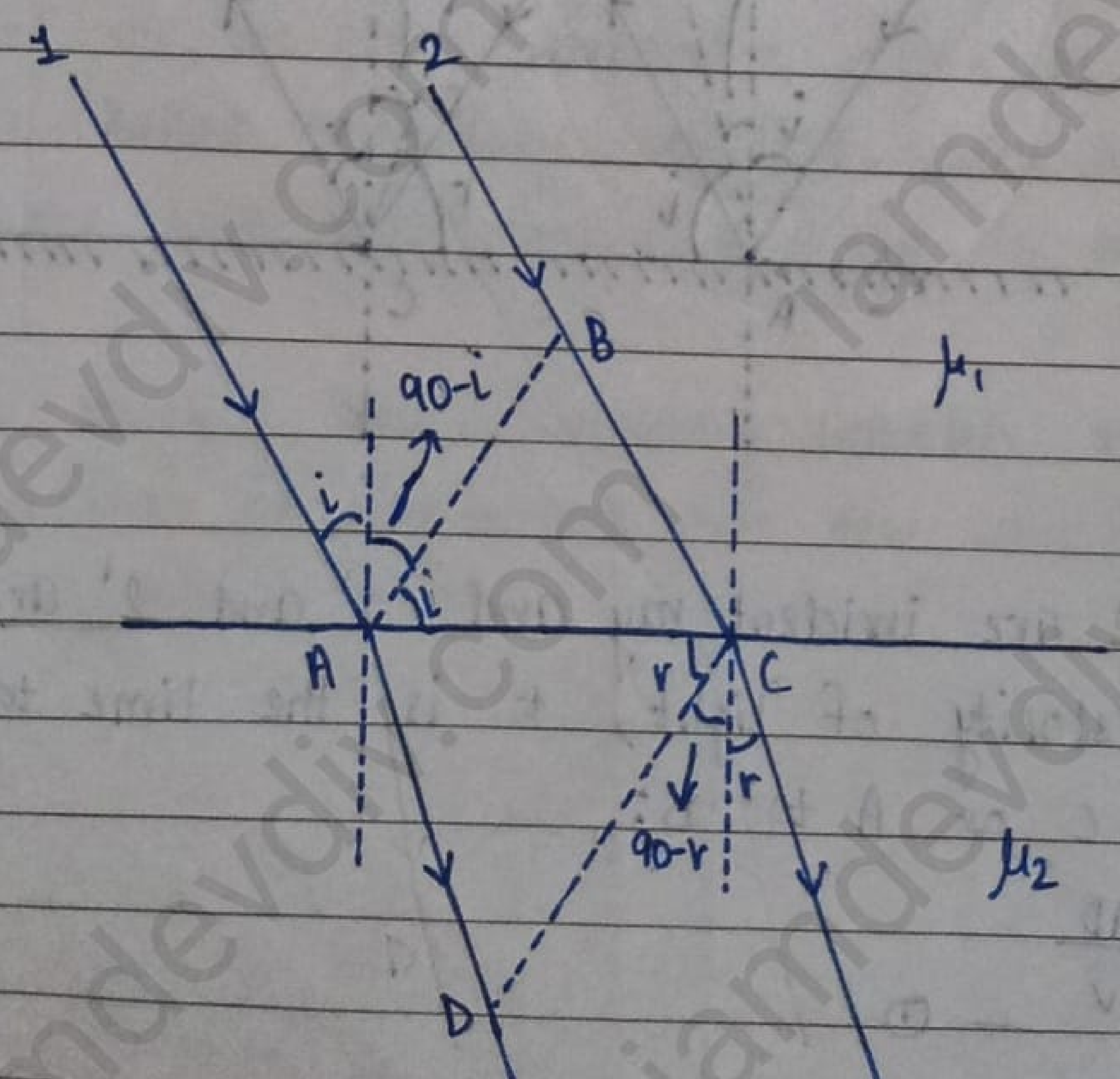
Put these values in ③,

$$AC \sin r = AC \sin i$$

$\Rightarrow \sin r = \sin i$

$\Rightarrow \boxed{i = r}$

☆ PROOF OF LAW OF REFRACTION ON THE BASIS OF WAVE THEORY



Time taken by the ray 2 to travel from B to C,

$$t = \frac{BC}{v_1}$$

Time taken by the ray 1 to travel from A to D,

$$t = \frac{AD}{v_2}$$

$$\frac{BC}{v_1} = \frac{AD}{v_2}$$

In ΔABC,

$$\sin i = \frac{BC}{AC} \Rightarrow BC = AC \sin i$$

In ΔADC,

$$\sin r = \frac{AD}{AC} \Rightarrow AD = AC \sin r$$

$$\frac{AC \sin i}{v_1} = \frac{AC \sin r}{v_2}$$

$$\frac{\sin i}{v_1} = \frac{\sin r}{v_2}$$

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$ [∵ v is inversly proportional to refractive index $v \propto \frac{1}{\mu}$]

$$\boxed{\frac{\sin i}{\sin r} = \mu}$$

★ SUPERPOSITION PRINCIPLE

When two or more waves travelling through a medium, super imposed on each other then a new wave is formed in which resultant displacement is equal to the vector sum of displacement due to individual waves.

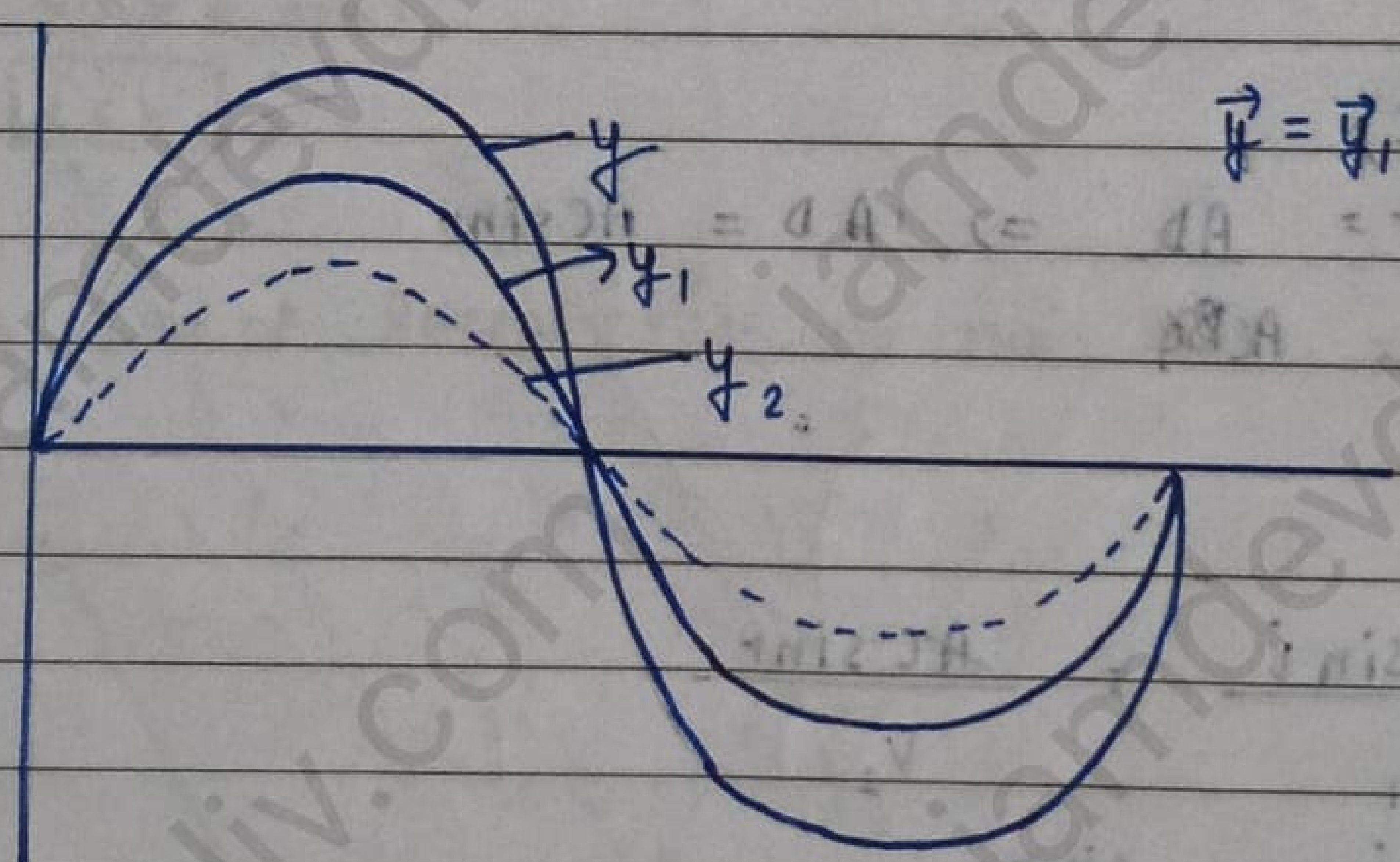
$$\vec{y} = \vec{y_1} + \vec{y_2} + \dots + \vec{y_n}$$

• INTERFERENCE OF LIGHT

The phenomena of redistribution of light energy in a medium on account of superposition of light wave from two coherent source.

* CONSTRUCTIVE INTERFERENCE

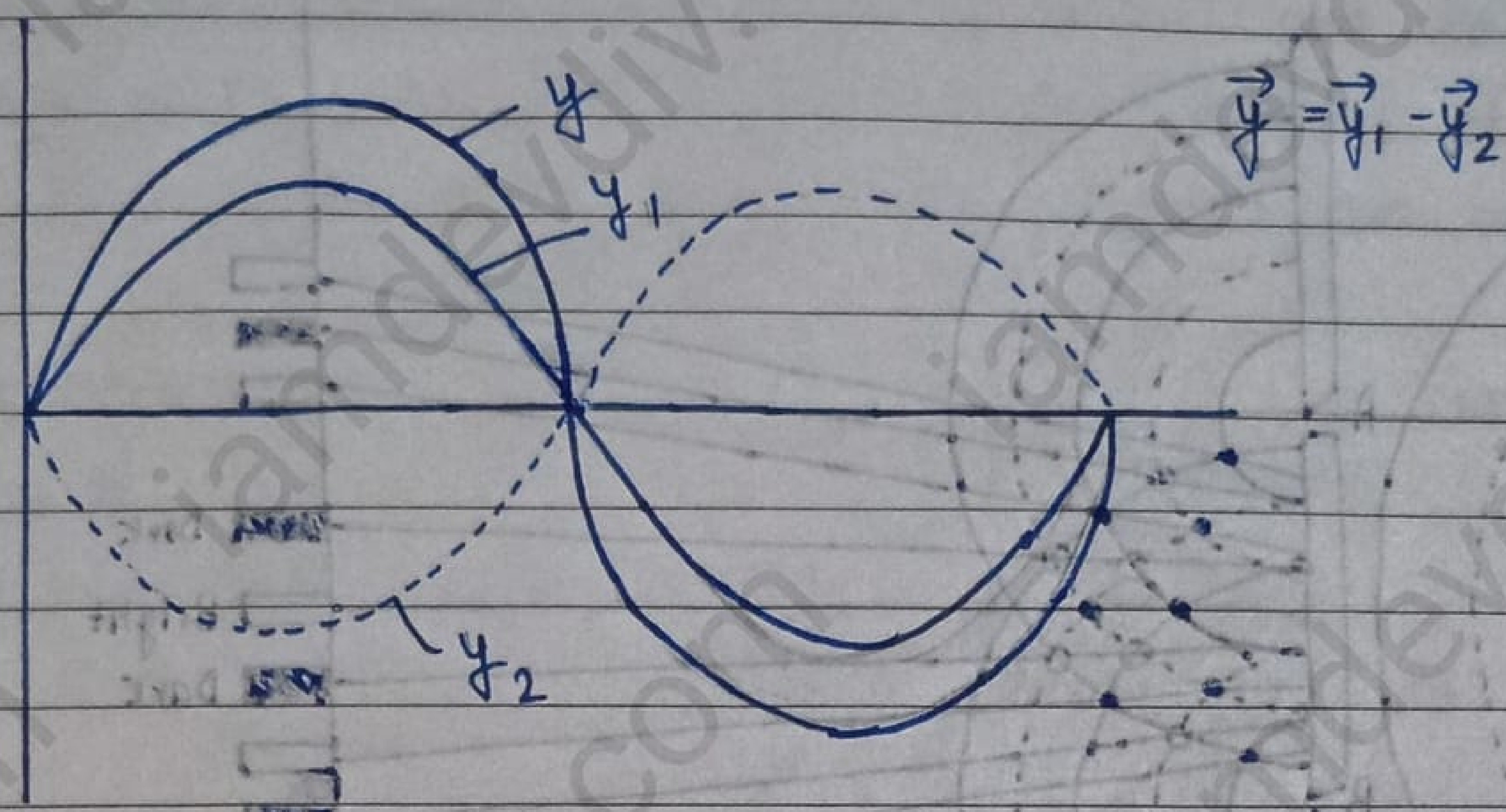
At the point where resultant intensity of light is maximum then interference is called constructive interference.



$$\vec{y} = \vec{y_1} + \vec{y_2}$$

* DESTRUCTIVE INTERFERENCE

At the point where resultant intensity of light is minimum then interference is called destructive interference.



- COHERENT SOURCE

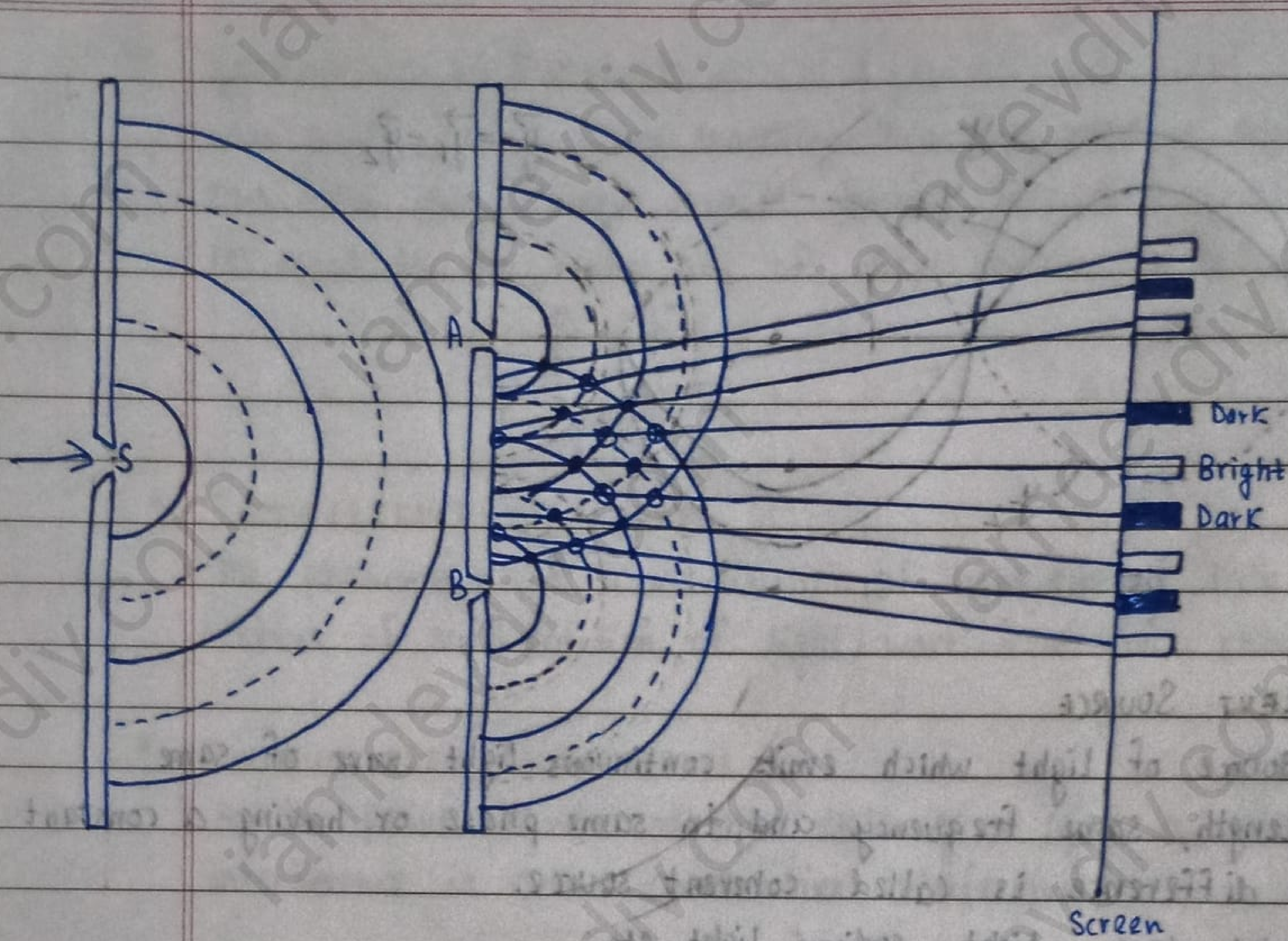
The source of light which emits continuous light wave of same wavelength, same frequency and in same phase or having a constant phase difference is called coherent source.
Example: Laser light, sodium light, etc.

* CONDITIONS FOR OBTAINING COHERENT SOURCE

1. Coherent source of light should be obtained from a single source by same device.
2. The two sources of light should ~~be~~ give monochromatic light i.e., light of same wavelength.
3. The path difference between the light waves from two sources should be small.

☆ YOUNG'S DOUBLE SLIT EXPERIMENT

S is a narrow slit illuminated by monochromatic source of light. At a suitable distance from S, there are two fine slits A and B placed symmetrically parallel to S. When a screen is placed at a large distance from the slits A and B then alternate bright and dark fringes appear on the screen.



- EXPLANATION

The appearance of bright and dark fringes can be explained on the basis of light. According to Huygen's principle, monochromatic source of light illuminating the slit as send spherical wave front. In this wavefront, solid arc represent crest and dotted arc represent trough. These wave front reach the slit A and B simultaneously which in turn become source of secondary wavelets. Thus, the two waves of same amplitude and same frequency with zero phase difference are given out by A and B. Dot represents constructive interference and cross represents destructive interference. When the pattern is observed on the screen, alternate bright and dark fringes were observed.

- EXPRESSION FOR FRINGE WIDTH

$$\beta = \frac{\lambda D}{d}$$

The distance between two successive bright and dark fringe is called fringe width where λ is wavelength, distance of screen from slit is D and ~~z~~ d is the distance between two coherent source.

☆ CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

~~$y_1 = a$~~

Let us consider light wave from two sources are

$$y_1 = a\sin\omega t$$
$$y_2 = b\sin(\omega t + \phi)$$

↳ phase difference

$$y = \vec{y_1} + \vec{y_2}$$
$$= a\sin\omega t + b\sin(\omega t + \phi)$$
$$= a\sin\omega t + b\sin\omega t\cos\phi + b\cos\omega t + \sin\phi$$
$$= \sin\omega t\,(a + b\cos\phi) + b\cos\omega t\sin\phi$$

Let $a + b\cos\phi = R\cos\theta$ — ①

and $b\sin\phi = R\sin\theta$ — ②

$$\Rightarrow y = \sin\omega t\, R\cos\theta + R\sin\theta\cos\omega t$$
$$\Rightarrow y = R\sin\omega t\cos\theta + R\sin\theta\cos\omega t$$
$$\Rightarrow y = R(\sin(\omega t + \theta))$$

↳ Resutant amplitude

$①^2 + ②^2$

$$(a + b\cos\phi)^2 + (b\sin\phi)^2 = R\cos^2\theta + R\sin^2\theta$$
$$\Rightarrow a^2 + b^2\cos^2\phi + 2ab\cos\phi + b^2\sin^2\phi = R^2(\cos^2\theta + \sin^2\theta)$$
$$\Rightarrow a^2 + b^2(\cos^2\phi + \sin^2\phi) + 2ab\cos\phi = R^2$$
$$\Rightarrow a^2 + b^2 + 2ab\cos\phi = R^2$$
$$\Rightarrow R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

R_{max} when $\cos\phi = 1$

$$\phi = 0, 2\pi, 4\pi, \ldots$$
$$= 2n\pi$$

$\Rightarrow R_{max} = \sqrt{a^2+b^2+2ab} = \sqrt{(a+b)^2} = a+b$

R_{min} when $\cos\phi = -1$

$\phi = \pi, 3\pi, 5\pi, \ldots$

$\phi = (2n+1)\pi$

$\Rightarrow R_{min} = \sqrt{a^2+b^2-2ab} = \sqrt{(a-b)^2} = a-b$

- IN CASE OF ~~INTERFEREN~~ INTENSITY

$I \propto (\text{Amplitude})^2$

$\Rightarrow I \propto (R)^2$

$\Rightarrow I = kR^2$

$\Rightarrow I = k(a^2+b^2+2ab\cos\phi)$

$\Rightarrow I = ka^2 + kb^2 + k2ab\cos\phi$

Let $ka^2 = I_1$ and $kb^2 = I_2$

$\sqrt{k}a = \sqrt{I_1}$ $\sqrt{k}b = \sqrt{I_2}$

$\Rightarrow I = I_1 + I_2 + 2\sqrt{k}a\sqrt{k}b\cos\phi$

$\Rightarrow I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}$

$I_{max} = (\sqrt{I_1}+\sqrt{I_2})^2$

Similarly, $I_{min} = (\sqrt{I_1}-\sqrt{I_2})^2$

★ RATIO OF INTENSITY AT MAXIMA AND MINIMA IN INTERFERENCE

$I \propto R^2$

$I = kR^2$

$I_{max} = kR^2_{max} = k(a+b)^2$

$I_{min} = kR^2_{min} = k(a-b)^2$

$$\frac{I_{max}}{I_{min}} = \frac{\cancel{k}(a+b)^2}{\cancel{k}(a-b)^2} = \frac{(a+b)^2}{(a-b)^2}$$

If W_1 and W_2 are width of two slits from which intensity of light I_1 and I_2 produce then

$$\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2}$$

where, a and b are amplitudes

★ CONDITIONS FOR SUSTAINED INTERFERENCE OF LIGHT

1. The two sources producing interference must be coherent.
2. The wave must be having same amplitude.
3. Two sources must be very close to each other.
4. The source must be monochromatic.
5. The two sources must be point sources.

Q What is the effect on interference fringes in young's double slit experiment (YDSE) due to each of following operation:

(a) Screen is moved away from the plane of slit.

(a) If the screen is moved away then D will increase and as per the formula $\left[\beta = \frac{\lambda D}{d}\right]$, fringe width will also increase. Therefore, pattern will be not so good.

(b) The monochromatic light is replaced by another monochromatic light of shorter wave length.

(b) If wavelength is decreased then β will also decrease, hence fringe width will also ~~increase~~ decrease. Therefore, more fringes will be observed and interference pattern will be good.

(c) The separation between two slits is increased.

(c) If d is increased, then fringe width (β) will decrease and more fringes will be observed.

(d) The source slit is moved closer to the double slit.

(d) When source is too close, the fringes get disappear and interference pattern become less and less sharp.

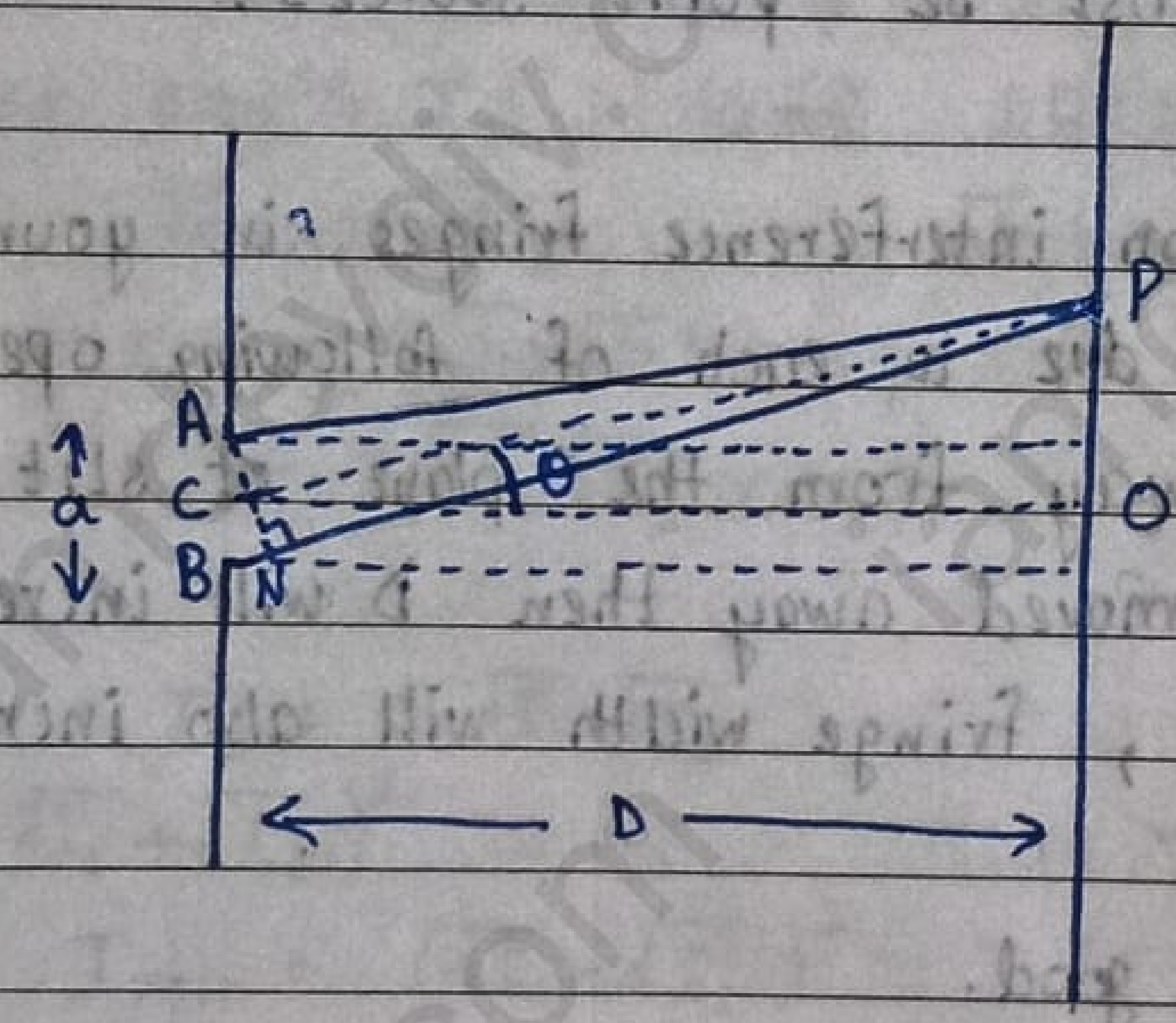
(e) The monochromatic source is replaced by source of white light.

(e) In this case, coloured fringes are observed.

★ DIFFRACTION ~~DEFLECTION~~ OF LIGHT

The phenomena of bending of light around the sharp corner of an object is called diffraction of light.

- DIFFRACTION OF LIGHT DUE TO SINGLE SLIT



~~Path difference ⇒ BP − AP~~

~~⇒ BN~~

~~In ΔANB~~,

~~$\sin\theta = \frac{BN}{a}$~~

~~$BN \Rightarrow a\sin\theta$~~

AB is an opening, O is a point on a screen at distance D such that waves from A to B reach at same distance, same phase

Let P is a point at elevation θ

Path difference $\Rightarrow BP - AP$

$\Rightarrow BN$

In ΔANB,

$$\sin\theta = \frac{BN}{a}$$

$$\Rightarrow BN = a\sin\theta$$

* CONDITION FOR SECONDARY MAXIMA

We observe that P is a bright point when path difference is $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$, $\frac{7\lambda}{2}$, ... $\cancel{(2n+1)\frac{\lambda}{2}}$

$$a\sin\theta = (2n+1)\frac{\lambda}{2}$$

where, $n = 1, 2, 3, \ldots$

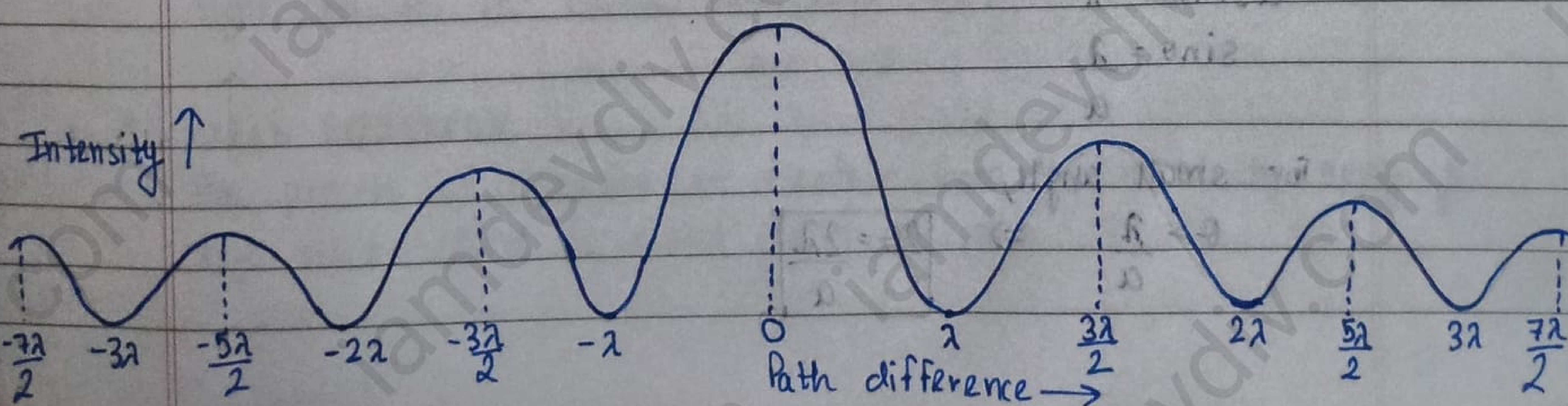
* CONDITION FOR SECONDARY MINIMA [DARK]

We observe that P is a dark point if ~~when~~ path difference is λ, 2λ, 3λ, ...

$$a\sin\theta = \lambda n$$

where, $n = 1, 2, 3, \ldots$

* GRAPH BETWEEN INTENSITY AND PATH DIFFERENCE



* WIDTH OF CENTRAL MAXIMA

The width of central maxima is the distance between first secondary minima on either sides of central bright fringe O. For first secondary minima,

$$a\sin\theta = n\lambda$$

For ~~fist~~ First sec. minima, $n = 1$

$$\Rightarrow \sin\theta = \frac{\lambda}{a} \quad \text{--- (1)}$$

In ΔPCO,

$$\sin\theta = \frac{y}{D} \quad \text{--- (2)}$$

Comparing (1) and (2),

$$\frac{\lambda}{a} = \frac{y}{D}$$

$$\Rightarrow y = \frac{\lambda D}{a}$$

$$\Rightarrow 2y = \frac{2\lambda D}{a}$$

$$\Rightarrow \boxed{W = \frac{2\lambda D}{a}}$$

* ANGULAR WIDTH OF CENTRAL MAXIMA

$$a\sin\theta = n\lambda$$

For $n = 1$,

$$a\sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{a}$$

For small angle,

$$\theta = \frac{\lambda}{a} \Rightarrow \boxed{2\theta = \frac{2\lambda}{a}}$$